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Application of the Maximum Flow Problem to Sensor Placement on Urban Road Networks for Homeland Security

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INTRODUCTION

Networks are essential components of our national infrastructure. However, those networks could be used by terrorists seeking to attack dense urban populations with weapons of mass destruction. In particular, large urban road networks provide many routes that terrorists could use to get close enough to a major city to make a harmful attack. One approach envisioned for protecting urban areas from such attack is to deploy (human-operated or fully automatic) sensors on the roads around cities to detect terrorists and their weapons so they can be stopped before they come within range of their targets. A key challenge to such an approach concerns how many sensors to buy and where to locate them. Indeed, the size and density of road networks would seem to make the cost of buying and operating these sensors prohibitive by requiring placement of sensors on hundreds if not thousands of road segments in order to protect any large city.

This challenge led to the work reported here, which shows that, contrary to first appearances, the number of sensors required to cover every possible route into a city is not prohibitively large. We apply graph theory to find a minimum cut set for a road network; i.e., to find a smallest set of road segments on which sensors must be placed to ensure that a terrorist traveling across the road network must encounter at least one sensor. We applied this theory to the actual road network of the New York City metropolitan area, and found that the minimum cut set was about 10^4 times smaller than the number of road segments in the network—the road network had approximately one million road segments and it yielded a minimum cut set of eighty-nine road segments. Thus, the minimum cut set problem for large urban road networks can be solved. Furthermore, the solution shows that the size of the cut set alone does not make it impractical to deploy a system of sensors that would cover all of the routes into the city on the road network.¹

The work reported here specifically concerns finding optimal locations for sensors for detecting terrorists, weapons, or other dangerous materials on roads leading into major cities. However, this work is generally applicable to finding minimum cut sets for any large network. It could be used to find optimal sensor locations on other transportation networks like railroads or subways. It could also be used to support offensive operations by locating a smallest set of segments in an adversary's network that would have to be cut in order to completely stop the flow through the network. Thus, the methodology presented here could have utility in other homeland security and military analyses.

There is considerable literature on graph theory, network optimization, and the minimum cut set problem. The references at the end of this article specifically address the minimum cut set problem. The accomplishments of the work reported here were a) to find and implement a practical way of solving large networks for minimum cut sets and b) to discover that the minimum cut set for a large U.S. urban road network was much smaller than what might have been expected given the number of road segments in the network.

ANALYTICAL PROBLEM

Terrorists seeking to attack a large city might use the roads leading into that city to transport personnel, weapons, or other dangerous materials. One approach for preventing such attacks is to deploy sensors on those roads to detect the movement and allow the interdiction of these entities before the terrorists reach their destination. To evaluate the feasibility of this approach, we would like to know the smallest number of sensors that must be deployed to ensure that a terrorist traveling into a city would encounter a sensor, and, of course, where to locate those sensors. Thus, we sought a methodology that could find a minimum cut set in a city's road network.²

The particular question addressed is as follows. Suppose an adversary has personnel, weapons, or other dangerous material at some distance from an urban area, and that the adversary can transport these on the roads of that area to move them into the interior of that area. Suppose sensors can be placed along the side of any road segment in that area. Then what is the minimum number of sensors needed, and where should these sensors be placed, so that a vehicle using any of the roads in the road network of that urban area must pass at least one sensor in going from any exterior location to any interior location? This paper explains the methodology and presents some results of applying that methodology to the New York City metropolitan area.

METHODOLOGY

If the road networks involved had been small enough, there would have been no methodological problem to solve. Solving for a minimum cut set is a well-known network problem, and existing map data and network solvers could have been used to answer the question posed above (see the references for discussions of this network problem). However, the road networks around major urban areas are very large; for example, the road network around New York City contains over one million road segments. Several, otherwise appropriate, network solvers cannot solve a problem this large.

New York is the largest and, perhaps, the most important city in the United States. It was attacked on September 11, 2001 and in 1993. Accordingly, a methodology that could not handle the New York region's road network would not be of interest, while one that could make it there would (likely) make it anywhere. Therefore, it was important that the methodology could be successfully applied to the New York region. So it was decided to test this methodology by identifying potential sensor locations—minimum cut sets—for the New York metropolitan area road network.

To analyze the road network around New York, we obtained a geographic information system database that contained all of the road segments in the U.S., where a road segment is defined as the portion of a road between consecutive intersections with other

roads. Thus, a road segment is a positive length of road rather than a point. (The length of these segments may provide some flexibility in locating sensors on a road network.) The database also includes some “false nodes” that are created when an otherwise straight road makes a turn that is not at an intersection. Such false nodes add to the size of the network but they do not affect the solution for the minimum cut set.

The database contains information that characterizes each road segment, both geographically and functionally, and it includes residential streets, urban alleyways, and other paths that are navigable vehicular routes. It is derived from hundreds of sources and is maintained and updated on a frequent basis. Thus, the database is perishable and must be updated (or the minimum cut sets must be otherwise verified) to ensure that the analysis produces valid results.

To obtain an accurate solution for a minimum cut set, one must have high quality data. Data that left out road segments, or contained errors regarding where the segments are connected, could give rise to false overall results. Assessing the quality of such a large database is difficult. The database that we used is a subset of the data from a commercially available product known as the “JServer” database. JServer data are normally accessed via custom applications to derive optimized trucking delivery routes and driver directions.

For this application, we assessed the quality of the data by comparing JServer data for Washington, D.C. streets against some known anomalies. In these visual comparisons, the data correctly identified blocked alleyways and traffic-circle entries and exits, which were sources of errors in other database products that we examined.

In addition, and more importantly, we subsequently evaluated our New York City cut sets visually and contextually against overhead images in Google Earth to see if there were any roads that bypassed the cut sets.³ This examination showed that our data and cut sets were remarkably consistent with the Google Earth overhead photographic raster imagery. Moreover, the compatibility of the database with the MapInfo Professional Geographic Information System allowed us to directly superimpose our minimum cut sets on the Google Earth imagery. This facilitated the manual examination and confirmation of the validity of the cut sets.

When we performed the Google Earth overhead imagery analysis to confirm the validity of the cut sets, we also looked to see where an adversary might be able to avoid road segments with sensors by driving off-road (e.g., through parking lots). We found about half a dozen locations where this might be possible. Thus, off-road routes do not appear to be a major factor in determining the number of sensors required. However, finding and blocking such routes to prevent an adversary from bypassing the sensors (as well as confirming the validity of the minimum cut set) could be important when designing a real urban road network sensor system.

The first step in this process of designing a sensor barrier is to determine where, in a general sense, the barrier is to be located. The methodology identifies a minimum cut set in the network, but one must first decide in what part of the network to put the minimum cut set. In our case, because we wanted a barrier surrounding the center of a city, we defined the barrier’s general location in terms of its distance from the center of the city. After determining the general area in which the barrier will be located, the second step is to apply the methodology to find a minimum cut set in that general location.

To determine where a minimum cut set was to be located, we drew two concentric circles around a central point in the urban area in question. The terrorists are assumed to start at an unknown (to us) location outside the outer circle, with the intention of reaching an unknown (to us) location inside the inner circle. The actual road network considered consists of those road segments that have at least one node (i.e., endpoint) between these two concentric circles.⁴

For this demonstration, the outer circle was a forty-five-mile radius circle centered at Times Square and the inner circle was a concentric fifteen-mile radius circle. Thus, the network considered is essentially contained in a thirty-mile-wide ring around Manhattan. Sensors on a minimum cut set within such a ring would allow an attack to be detected and potentially interdicted at least fifteen miles from Times Square. A ring of such width encompasses a large road network and so gives the network solver the potential to find a small minimum cut set. Figures 1 and 2 below depict these fifteen- and forty-five-mile circular boundary lines and the road segments contained in the corresponding ring between them.



Figure 1. New York City region road segments and the forty-five- and fifteen-mile boundaries

The ring shown in Figure 2 contains 488,951 road segments, 414,640 of which are from two-way road segments (which we converted into two one-way segments) and the other 74,311 are from one-way road segments. There are 722 road segments that cross the outer 45-mile radius boundary and 708 segments that cross the inner fifteen-mile radius boundary.

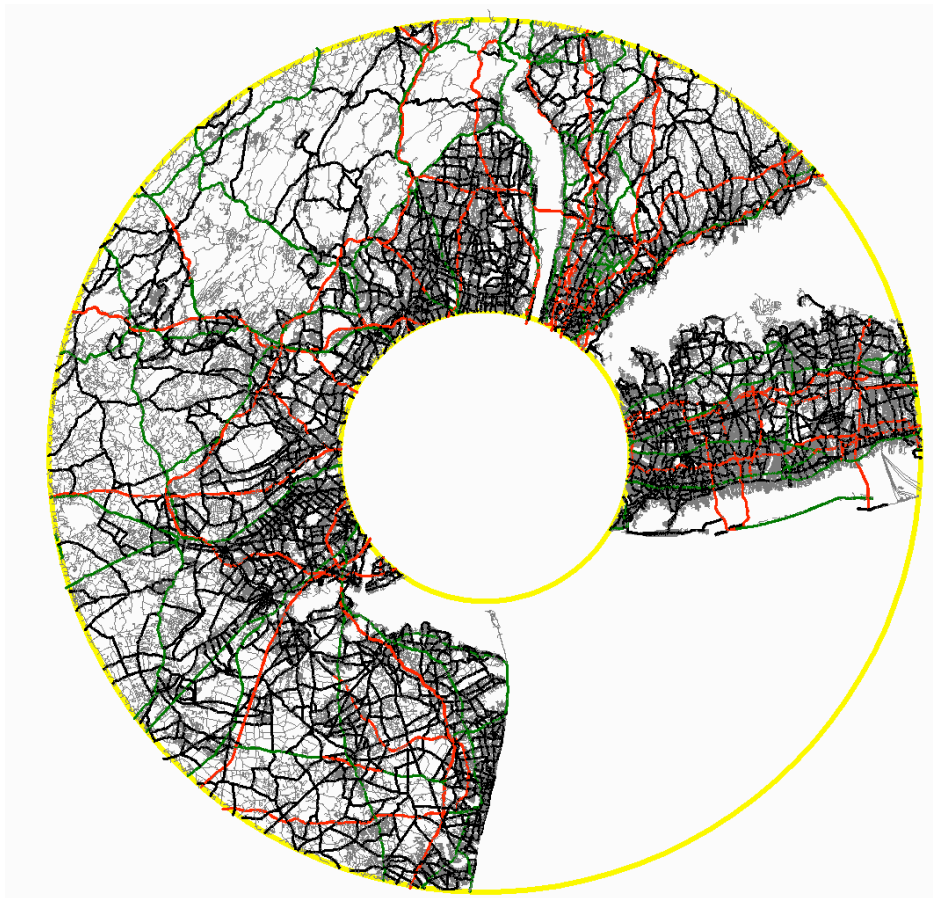


Figure 2. New York City region road segments between the forty-five- and fifteen-mile boundaries

It is important to note that these boundaries can be selected in any reasonable manner desired and they can be of any reasonable size or shape. The choice to use circles here is not a restriction of the methodology but rather is a choice of convenience for this initial investigatory analysis. The boundaries we used in our example would be suitable for designing a sensor system to protect the high population density areas around New York City. However, one could use boundaries closer in (and of a conforming shape) to design a system to protect a smaller area, e.g., Manhattan, or boundaries farther out to protect a larger area, e.g., New York and Philadelphia.

It is interesting to note (Figure 1) that the density of the road network decreases as one moves away from the cities. Thus, the minimum cut set for a larger region might be

smaller than the minimum cut set for a smaller region (despite the larger region's longer perimeter) if the larger region extended into the low-density portion of the road network. Minimum cut sets that we have prepared for the New York region using fifteen-mile wide rings subsequent to the analysis reported here have demonstrated that property. The minimum cut set can also be smaller if the network boundaries are drawn so as to include areas where the road density is limited by the presence of natural barriers like rivers. For example, a minimum cut set for Manhattan Island would consist of the limited number of bridges and tunnels connecting the island to the surrounding areas.

An interesting and somewhat counterintuitive property of these boundaries is that, as a ring is expanded (by increasing the outer radius, decreasing the inner radius, or both), the minimum cut set can get smaller but not larger. To see this, imagine a narrower ring A whose inner and outer boundaries lie between the inner and outer boundaries of a wider ring B. Every cut set for ring A must also be a cut set for ring B, but not vice-versa. Thus, the minimum cut set for ring B can be no larger than the minimum cut set for ring A. It is also possible that a cut set for ring B will be smaller than a minimum cut set for ring A. Thus, a minimum cut set for ring B must be as small as, or smaller than, a minimum cut set for ring A.

Figure 3 gives a less detailed map of the fifteen-mile-radius disk of Figure 1—but it shows relevant city names and major route numbers. Figure 4 gives a less detailed map of the forty-five-mile disk—but it also shows relevant city names and major route numbers. Figures 3 and 4 are not drawn to the same scale as each other or to the scale of Figures 1, 2, 5, and 6.

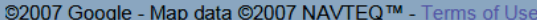


Figure 3. A less detailed map of the fifteen-mile-radius disk given in Figure 1

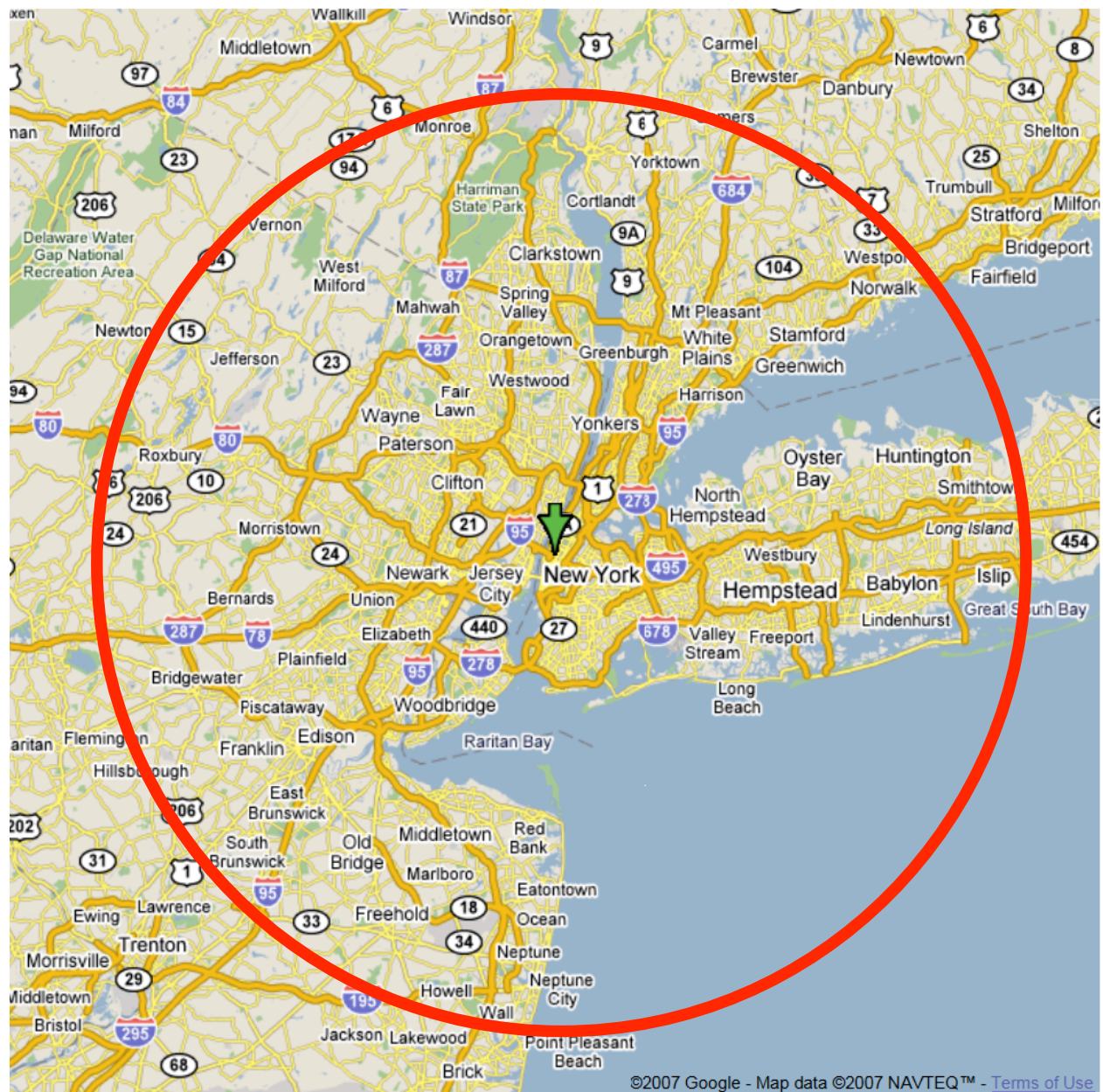


Figure 4. A less detailed map of the forty-five-mile-radius disk given on Figure 2

The second step in our process was to find a minimum cut set for the road network whose segments have one or both nodes between these boundary lines. This is a well-known problem in graph theory, and it might be expected that there would be many solvers that could be used to obtain a solution to it. However, all but one of the solvers we considered could not find minimum cut sets in networks as large as the road network surrounding New York City. One, the GNET solver, could do so. Several solvers were Excel-based, and so were restricted by the size limitations of Excel. Others could only accept the problem in the form of a general linear program and their LP-interfaces were unable to handle the problem. As a result, it took significant effort for us to identify just one solver that could find minimum cut sets in networks as large as the road network surrounding New York City. That one was the GNET solver.

GNET is a proprietary network solver that runs on an Intel-based PC under MS Windows.⁵ (See G.H. Bradley, et al. [1977] for a description of GNET's theoretical basis.) GNET generates a minimum cut set when given a mathematical description of a network, which was obtained from JServer. GNET is designed to handle very large networks, and our experience so far is that it can handle networks of over one million arcs.

To let the max-flow algorithm find a minimal cut set, the following structure was used. Each of the segments with both nodes inside the ring was given a capacity of 1. An artificial "super-source" node was added outside the outer ring, and an artificial "super-sink" node was added inside the inner ring. The outer node of each segment crossing the outer ring was changed to this super-source node and these segments were given an infinite capacity. Similarly, the inner node of each segment crossing the inner ring was changed to this super-sink node, and these segments were also given an infinite capacity. Finally, an artificial road segment going from the super-sink to the super-source was added, also with an infinite capacity.

Some network analyses are concerned with the different capacities of the individual arcs in the network because they seek to ascertain the effect of cutting sets of arcs on the network capacity or the flow through individual arcs in the network. In our case, while real roads have different traffic capacities, we assigned all real road segments a capacity of 1 because we sought to find the minimum cut set that would block (cover) all paths through the network.

Finding a minimum cut set requires preparing the road network data in the manner described here. This would be easy to do with ninety road segments, but is less so with 900,000. To re-structure the network into nodes and one-way arcs, we converted each of the 414,640 two-way road segments between two nodes into two segments running in opposite directions between those two nodes. We also created one super-source node and one super-sink node. This yielded a total of 903,591 road segments with both of their nodes inside of the ring on Figure 2 plus 1,430 boundary-crossing road segments. Accounting for all of these road segments, plus the super-sink to super-source segment, produced a total of 905,022 road segments.

We then solved this network for its maximum flow and, hence, for a minimum cut set. This minimum cut set gives the smallest number of sensors that must be deployed in order to ensure that any vehicle attempting to penetrate the inner (fifteen-mile) circle, starting on any road from outside of the outer (forty-five-mile) circle, will necessarily encounter at least one sensor. It should be noted that there could be more than one

minimum cut set, and if there is more than one, the cut sets may or may not include some of the same road segments.

In other applications, one might not only be concerned about transportation into a city, but also about transportation from points within the city to points outside the city, and from points within the city to other points within the city. This methodology has the tested capability to find minimum cut sets for outward transportation as well as inward. It should be noted that neither the real New York City road network nor our representation of it, is symmetric. One-way streets and highways can make the number of routes available to travel inward different from the number available to travel outward. Thus, there can be differences between the solution to the forward (outside-to-inside) problem and the solution to the reverse (inside-to-outside) problem. We demonstrate a solution to the outward transportation problem below.

RESULTS

Figure 5 shows a minimum cut set for the network described above. It contains eighty-nine road segments. Figure 5 is too coarse to identify the particular road segments that constitute this cut set. The precise identity of these road segments is contained in the output data files produced in the network analysis.

We believe that the results of this analysis are quite surprising. In particular, the result that only eighty-nine sensor locations are required to cover every possible vehicular route into the fifteen-mile disk around Times Square is unexpected and counterintuitive. That number is about one one-hundredth of one percent of the 829,820 road segments in the network (not counting the notional source-connecting and sink-connecting segments). It is also about 10 percent of the number of road segments that cross the outer forty-five-mile radius boundary (722) or the inner fifteen-mile radius boundary (708). Therefore, building a sensor barrier around New York City on a minimum cut set would be considerably more efficient than simply placing a sensor on each road segment that crossed the inner boundary or that crossed the outer boundary of the network. Thus, while building a road network sensor barrier around New York City might have initially appeared to be impractical, the eighty-nine-segment cut set shows that this is not necessarily so.

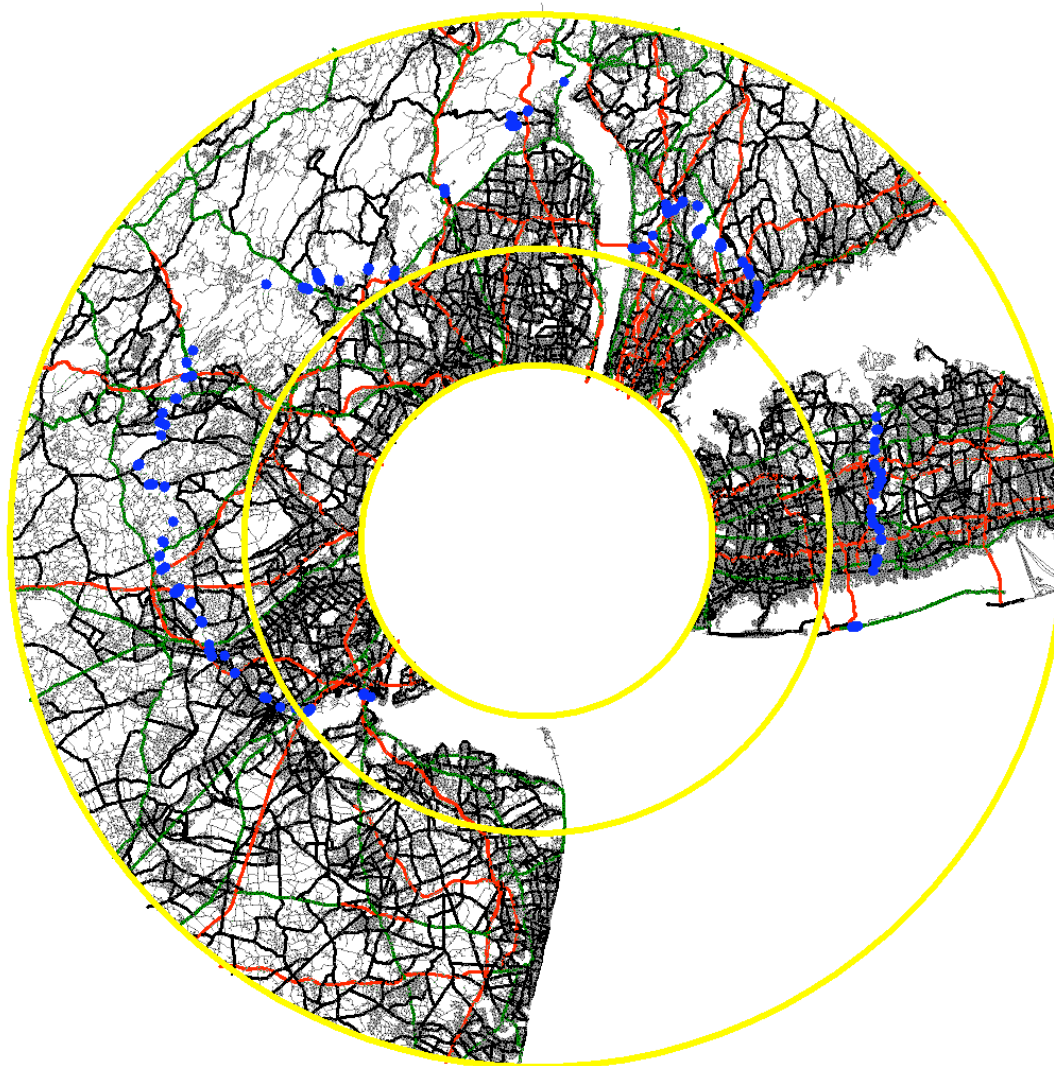


Figure 5. New York region road segments between the forty-five- and fifteen-mile radii circles showing the eighty-nine minimum cut-set segments, and a twenty-five-mile intermediate radius circle.

Another observation one can make about the road network surrounding New York City is that, as depicted in Figure 5, eighty-six of the eighty-nine cut set segments are outside a twenty-five-mile intermediate radius. Accordingly, if sensors were deployed on this cut set, and if the adversary were to choose a route at random such that encountering any sensor was equally as likely as encountering any other sensor, then it would be twenty-eight times more likely for the encounter to occur more than twenty-five miles from Times Square than within twenty-five miles of Times Square.

As stated above, our methodology is able to solve for outward as well as inward transportation. As a demonstration, we solved for a minimum cut set for the same area (the ring between fifteen and forty-five miles from Times Square) by assuming that the vehicle would begin inside the inner ring and move to a point outside the outer ring. Figure 6 depicts both the inward (blue) and outward (pink) minimum cuts sets that were generated in this test.

There are eighty-nine road segments in both the inbound and outbound minimum cut sets. If the road network were symmetric (which it is not), then an inbound minimum cut set would always be paired with a corresponding outbound minimum cut set. However, for asymmetric networks, there need not be any such relationship.

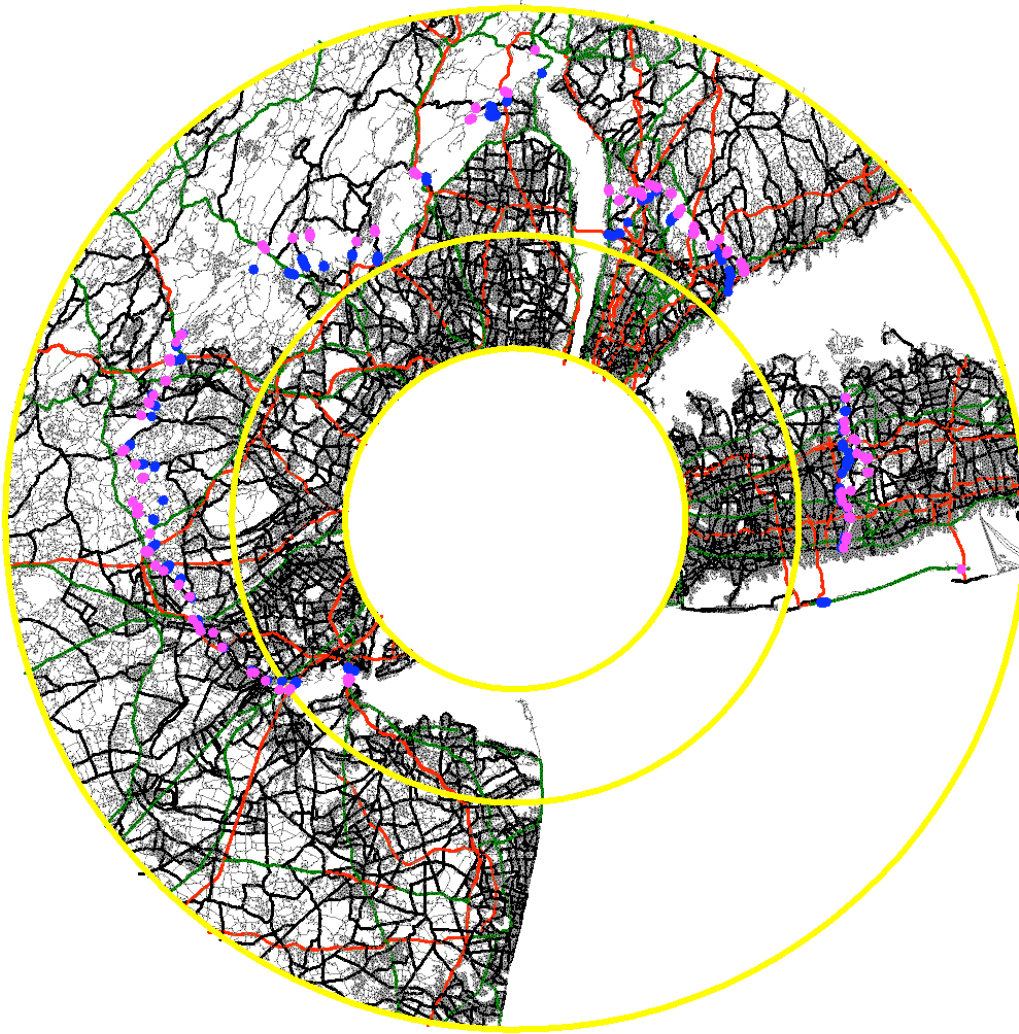


Figure 6. Forward and reverse flow minimum cut sets for forty-five to fifteen mile radii

This paper demonstrates the ability to find a minimum cut set for a road network containing over one million segments. Accordingly, there is good reason to believe that a minimum cut set around any single population center in United States can be found. However, it is possible that one might need to analyze larger networks to identify minimum cut sets that encompass multiple cities or regions. GNET's developers think that GNET could handle networks as large as four million arcs. But beyond that size, more memory and computer CPU power will be required than is currently available on single core PCs (like those that were used here). Nevertheless, one might be able to analyze entire regions of the United States with our existing arrangement of hardware and software by judiciously selecting the network boundaries.

BARRIERS WITH FEWER SENSORS

Until now, we have assumed that the number of sensors deployed would be equal to the number of road segments in a minimum cut set. However, one might wish to build a defensive system (barrier) with fewer sensors than cut set segments. If we have no knowledge of which route a terrorist may take in trying to traverse the barrier, and if the terrorist has no advance knowledge of where we will place our sensors, then the optimal approach is for us to deploy our sensors randomly (uniformly and independently) across the segments of a minimum cut set, with no more than one sensor per cut set segment. This would yield a probability of the terrorist encountering a sensor equal to the number of sensors divided by the number of segments in the minimum cut set. Such a system would provide some level of protection and might be sufficient to deter an attack.

The assumptions underlying this result are important. If the terrorists knew the locations of our sensors ahead of time (and we had fewer sensors than cut set segments), then they would simply choose an undefended route and avoid all of the sensors. On the other hand, if we knew that the terrorists had preferences for taking certain routes over others, independent of our deployment of sensors, then we could place our sensors to cover those routes to maximize the probability of an encounter with a sensor. These possibilities suggest that we should consider concealing or frequently relocating our sensors so that terrorists will not know where they are.

NEXT STEPS

We found that a minimum cut set for New York City's road network contains many fewer segments than the network as a whole, and fewer even than the number of segments that are intersected by large circles drawn around the city. But New York is hardly a typical city. It could be worthwhile to apply this methodology to other cities to see if their minimum cut sets exhibited similar characteristics. Different road network locations and layouts might cause minimum cut sets to be relatively larger or smaller than they are for New York City.

Our results for New York City suggest that this methodology could be a useful tool for designing a system of terrorism countermeasures on an urban road network (or some other transportation network, e.g., rail or subway). However, solving for a minimum cut set is only one step in that process. After a cut set is found, it needs to be checked for ways that an adversary could bypass the cut segments (i.e., sensor locations) by driving off road or by using roads not shown on the map. When designing a real system, bypass possibilities and constraints on constructing, operating, and servicing equipment should be considered when selecting sensor locations.

After identifying potential sensor locations, system-level cost-effectiveness analyses (with an appropriate cost model) or risk management analyses could determine how many sensors to acquire and deploy. Analyses could address the performance of the sensors in terms of the rates of false negatives and false positives for different entities (terrorists, weapons, materials) as well as the effect of false positives on urban area traffic. They might address the vulnerability of the sensor system to scouting and spoofing by the adversary. They might address the availability and capabilities of interdicting forces to stop vehicles that give positive responses without allowing terrorists to release their weapons. Analyses might consider the potential for an effective countermeasure system to deter an adversary from attacking the city in the first place.

IDA has addressed these issues in other research it has performed. Nevertheless, the methodology discussed in this article should be valuable for enabling the design of urban road network sensor systems by solving the urban road network minimum cut set problem.

As a final note, this work has only addressed the location of sensors on roads. Research addressing the more general problem of protecting geographic areas from attack would also consider detecting and interdicting terrorists on alternative attack pathways employing air, water, or off-road land transportation as relevant.

SUMMARY

We have developed a methodology to help find optimal locations for sensors for detecting entities or materials transported on roadways around urban regions in the United States. The methodology uses graph theory to solve the maximum flow problem and identify a minimum cut set in networks containing over one million road segments. We applied the methodology to the road network of the New York City metropolitan area and found that, for a ring between fifteen and forty-five miles from Times Square, the minimum cut set contained only eighty-nine segments. This methodology and analysis is significant for two reasons. First, to our knowledge, networks as large as the road network around New York have not previously been analyzed for minimum cut sets. Second, the minimum cut set we found is much smaller than what one might have expected from the number of road segments in the network.

This methodology is potentially broadly applicable. It could be used to find optimal locations for sensors intended to detect, or defenses intended to interdict, any materials or entities on roads or on any other network. It could also be used to support offensive operations by locating the smallest set of segments in an adversary's network to cut in order to completely stop flow through that network. Thus, this methodology could have utility in other homeland security and military analyses.

The authors are research staff members at the Institute for Defense Analyses. IDA is a federally-funded research and development center that assists the Department of Defense and other agencies in addressing important national security issues, particularly those requiring scientific and technical expertise. Dr. Robert Atwell received degrees from Manhattan College and Georgetown University. He joined IDA in 1984. Dr. Lowell Bruce Anderson received degrees from Johns Hopkins and Northwestern, served in Vietnam, and joined IDA in 1971. He has won the ORSA/MAS Prize and IDA's Goodpaster Award. Robert Bovey and Sean Barnett were task leads on this effort. Dr. Bovey has specialized for the last ten years in homeland security and homeland defense subjects. Dr. Barnett's work has focused on military force structure and mobilization planning, risk analysis, and combat modeling. The research presented in this article was sponsored by the Department of Homeland Security. The authors may be contacted via Dr. Barnett, dbarnett@ida.org.

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¹ Guaranteeing an encounter with a sensor is only the first step in designing a system; it does not guarantee that the terrorist or his weapons would be detected or interdicted. As we discuss briefly at the end of the article, in designing a sensor system to be deployed around a city, one would also consider the ability of sensors to detect, and response forces to interdict, the terrorists who encountered the sensors.

² An alternative to placing sensors would be to simply block roads (permanently or temporarily). Placing a sensor or a roadblock on each of the minimum cut set road segments would ensure that an adversary would encounter either a sensor or a blocked road en route to the interior of the urban region. To simplify the discussion, henceforth the article speaks in terms of placing sensors rather than roadblocks.

³ The overhead imagery analysis was performed at IDA by Adam Mulliken and Robert Kraig.

⁴ This assumes that there is no single road segment that has one node outside the outer boundary and one inside the inner boundary. However, it would be easy to handle such road segments, if, in future cases, any were to exist.

⁵ Insight, Inc., Manassas, VA, copyright 1975, 1982, 1999, and 2006.